Tomography: A Geometric and Computational Approach

CT scan of a brain skull colored for clarity. (Wikipedia Commons)

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What is computational thinking?
Computational thinking is a high level thought process that considers the world in computational terms. It begins with learning to see opportunities to compute something, and it develops to include such considerations as computational complexity, utility of approximate solutions, computational resource implications of different algorithms, selection of appropriate data structures and the ease of coding, maintaining, and using the resulting program. Computational thinking is applicable across disciplinary domains because it takes place at a level of abstraction where similarities and differences can be seen in terms of the computational strategies available. A person skilled in computational thinking is able to harness the power of computing to gain insights. At its best, computational thinking is multi-disciplinary and cross-disciplinary thinking with an emphasis on the benefits of computational strategies to augment human insights. Computational thinking is a way of looking at the world in terms of how information can be generated, related, analyzed, represented, and shared.

Computational thinking in this module
- Describe the problem you want to solve;
- Logically organize and obtain data;
- Analyze the problem, think of different ways to solve it, and think of ways to use computers to help solve it - think outside the box;
- Use abstraction, estimates, and simplifying assumptions to formulate a model to help solve the problem;
- Use computer simulation and spreadsheets as tools;
- Implement various solutions with the goal of achieving the most effective, and most efficient solution;
- Construct ways to handle uncertainty in the solution to increase your confidence in the solution.

Types of classes where this module can be used:
This module could be used in a number of different classes in grades 10-14, many of which may have the flexibility in curricula to incorporate non-standard materials. Examples of potential target classrooms include those that discuss uses of technology. These could include computer science, biology, mathematics, or health classes.
Summary of the Module
The purpose of the module is to provide a general background on computer-generated tomography and study how CT scan images are created using 3-D reconstruction of objects using 2-dimensional pieces (slices) of the object. CT scans are currently used in medical imaging, archeology, food safety, structural integrity, virtual autopsies, and in many other applications.

The Main questions of the module are: How can 3-D images be created from 2-D images (of pieces) of the object? How much computational power and what computational thinking skills are required to do these reconstructions and what does this reconstruction depend on?

Teacher notes are all in red.

Section 1: Introduction to Tomography from a Computational Viewpoint
Driving Question: What are CT scans and how are they performed?
This section introduces imaging techniques and some of the applications to food safety and structural soundness along with the medical applications. Activities use probes of a ball of yarn and student hands to illustrate in a simple way what happens when CT scans are taken of the human body or other structure.

Today it may be difficult for us to imagine how mysterious the inside of a living person seemed more than 100 years ago when x-rays were discovered in 1895. Since 1895, there have been several advances in x-rays – not only in medical imaging. Most people have had dental x-rays looking for cavities or an x-ray of an arm or leg verifying an injury. Currently x-rays are also used to see inside baggage in airports or to see inside trucks crossing the border. Imaging techniques have advanced greatly, and today we have CT and PET scans to help visualize the interior of humans. How do CT and PET scans work, and what is tomography? Tomography is the science of examining an internal structure with external measurements. Literally, tomography means “slice imaging”. To help understand what tomography is, we begin with an activity:

Activity 1: Probing a ball of yarn
Take a loosely wound ball of yarn and put an eraser (or other object) into the middle of it without the students seeing you do it (ahead of time is best). Embedding an object should be is possible with most balls of large weight yarn. (Equivalently you can wrap yarn loosely around the object). Pass the yarn around the classroom. Students should feel the weight of the yarn and answer the following questions:
1. Does it feel heavier than you think it should feel?
2. What are possible explanations for why it appears heavier?
   With a knitting needle or sharp pencil (no unraveling), probe the ball in all directions.
3. When you probe the ball, what do you feel?
4. Can you tell if there is anything solid in the ball of yarn? How do you know?
5. What can you figure out about the object inside the yarn?
6. What measurements could you record to make the solution more precise?
7. Does it make a difference how big the object is when you are searching for it?

By probing the ball, students can determine if there is something inside the ball and they can get an idea of the shape and size of the object inside the ball, without having to unravel the yarn. Here are sample answers to the questions:

1. It may or may not feel heavier depending on the object you place inside. We recommend placing something heavier so students can feel the object’s presence.
2. There may be something inside.
3. The needle will get stopped when it hits the inserted object, and the needle should go all the way through when it misses the object.
4. You can tell there is something in it when the needle gets blocked.
5. By probing from many different angles and feeling how far in the needle goes, we can get a guess of its size and location. If we’re really careful, we might even be able to guess the shape.
6. We could measure how far the needle goes and record the angles. We could also weigh other yarn to get an idea of the extra weight of the object.
7. Absolutely. The smaller (and lighter!) the object, the harder it is to find.

A CT scan hopes to achieve the same result as probing with the knitting needle, but more accurately. Instead of collecting information by probing, x-rays (CT-scans) collect information on how much energy is absorbed as the body is scanned and this is recorded in the computer. Figure 1 shows the inside of a CT scanner, where T is the transmitter, D is the detector, X are the x-rays, and R represents the rotation direction.
Figure 1: A modern (2006) CT scanner with the cover removed, demonstrating the principle of operation. The X-ray tube and the detectors are mounted on a ring shaped gantry. The patient lies in the center of the ring while the gantry rotates around them. (From Wikipedia Commons.)

**Homework for Section 1:**
1. What kind of data is collected from medical CT scans: How safe is it?
2. What are two questions that you have about CT scans?
3. What other types of tomography are there? Use the web and research one other type of tomography (see list below) and give a one-page summary that answers the following questions. Be prepared to give a 3-5 minute presentation the next day in class:
   a. What type of tomography did you choose?
   b. What is it used for?
   c. What are its advantages? Disadvantages?
   d. What is a nontechnical description of how it works?
   e. How is it similar and different from the yarn example? The CT example?

Possible types of tomography:
1. Pipeline tomography
2. Magnetic resonance imaging (MRI)
3. Functional MRI
4. Electrical impedance tomography
5. Atom-probe tomography
6. Optical tomography
7. Ocean acoustic tomography
8. Food-safety tomography
9. Concrete-solidity tomography

Selected Solutions for Section 1 Homework

1. CT scans collect data on the amount of x-ray absorption. Because CT scans are thousands of x-rays, it is not recommended that you have many of them.

2. Example
   b. It identifies defects in pipelines
   c. Advantages – Do not have to dig up pipelines or use PIG (pipeline inspection gauges)
      Disadvantages – Cannot guarantee the detection of defects that do not cause a change in the level of stress deformed condition – i.e. blow holes, pitting.
   d. It uses a magnet.
   e. The magnet plays the role of the knitting needle or x-ray beam.
      (Perhaps.)

You can use the video at any time during the two days, but the beginning of day 2 might be ideal.
Section 2: Introduction to Reconstructions

Driving Question: Given the “shape” from different views, how can we put together a 3-D object? How do we determine the internal structure of a 3-D object?

Let’s start with an activity that focuses on determining the internal structure of a 2-D object before approaching 3-D objects.

Activity 2: Grid Cube Activity
For this activity, each group will need a 5 by 5 board (included in appendix) and 3-5 pennies. Here is an example of the activity: Given the numbered grid in Figure 5, figure out where the pennies are. To understand the game, we first need to understand what the numbers on a grid represent. Let’s consider the grid with 3 pennies shown in Figure 2. We are going to assign a number to each cube (box or cell) on the grid. Four lines pass through each cube on the grid: horizontal, vertical, and two diagonals. Count the number of pennies on each of the four lines passing through that cube – the sum of those four numbers is the number for that cube.

Figure 2: Three pennies on a board and the representation of those three pennies in a grid.

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1 Adapated from Mathematical News – Alan Adolphson – www.math.okstate.edu
Figures 3 and 4 provide examples of the four lines through cubes on a grid.

Let’s begin to determine the numbers for each cube starting with the empty first cube in the first row/first column as shown in Figure 4:

Step 1: The vertical line through the cube passes through no pennies, for a count of 0.
Step 2: The horizontal line through the cube passes through no pennies, for a count of 0.
Step 3: The diagonal going up from left to right through the cube passes through no pennies, for a count of 0.
Step 4: The diagonal going down from left to right through the cube passes through two pennies, for a count of 2.
Step 5: Add up all the counts, for a total count of 2.

Now, consider a cube with a penny on it – (second row, second column) as indicated in Figure 3:

Step 1: The vertical line through the cube passes through 2 pennies, for a count of 2.
Step 2: The horizontal line through the cube passes through 1 penny, for a count of 1.
Step 3: The diagonal going up from left to right through the cube passes through 1 penny, for a count of 1.
Step 4: The diagonal going down from left to right through the cube passes through two pennies, for a count of 2.
Step 5: Add up all the counts, for a total count of $2+1+1+2 = 6$.
The complete numbered grid is shown in Figure 5.

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Figure 5: The numbered grid corresponding to Figure 2.

Try some new ones: Given the grids in figure 6, find the corresponding number boards.

Figure 6: New grids with 3 and 4 pennies.

Solutions:

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The previous instructions and problems were to help you understand where the numbers come from. The real challenge is going the other way. That is, we need to go from a numbered grid to the location of the pennies.
Questions:
1. Given a numbered grid with three hidden pennies, can you tell where the three pennies are? Can you guess a general strategy?
Solution: For a 5 by 5 grid and 3 pennies, the location of the pennies is where the three highest numbers are. Notice that a square with a penny will correspond to at least a 4 because four lines go through it. This implies that any number less than 4 cannot have a coin in that spot.

2. Given a numbered grid with four hidden pennies, can you tell where the four pennies are? Can you think of a scheme to help eliminate most possibilities?
Solution: For a 5 by 5 grid and 4 pennies, the location of the pennies is where the three highest numbers are. However, as in the example above, there may be more than 4 highest numbers, but that narrows down the possibilities. Then, you can try each of the possibilities to see if any match the numbers that appear. One strategy is to assume there is a coin on the highest number, then reduce by one the numbers in each row, column and both diagonals that intersect that coin. Then repeat the process. For example:

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3. Given the numbered grids in Figure 7, find the location of three or four pennies.
Figure 7: New numbered grids. Each with either 3 or 4 pennies.

Solutions

4. **Challenge question**: At this point, we have used a 5 by 5 grid with 3 and 4 pennies. Given a 5 by 5 number grid that has 5 pennies, will your same strategies work? What about 6 or more pennies for a 5 by 5 grid? What happens if the grids are 6 by 6 or larger?

**Activity 3**: Milk Bottle Activity²
Imagine that milk and fruit juice are delivered in bottles that are placed in trays with 9 compartments arranged as a 3 by 3 grid, stacked one on top of another. Each compartment of the tray contains a bottle, which may contain milk, juice, or may be empty. The goal is to try to figure out in a given tray which bottle is in which compartment. Since other trays are on top, we cannot just look down on top of the tray.

**Computational Thinking Alert**: Describe the problem you want to solve and think about ways to solve the problem.

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² Saving Lives: The Mathematics of Tomography, Chris Budd & Cathryn Mitchell
By looking through the sides we can see how much light (energy) has been absorbed in different directions. Different types of bottles absorb different amounts of light. Assume that milk bottles absorb 3 units, juice bottles 2 units and empty bottles 1 unit. If a light beam is shone through several bottles, then the absorptions of each bottle add up to a total absorption, which we can detect. For example, if a light beam shines through a juice bottle, then it shines through an empty bottle, and finally it shines through another empty bottle, then 4 units are absorbed.

**Computational Thinking Alert:** Logically organize your information and obtain more data as needed.

Figure 9 indicates the total amount of light absorbed when shining light through each of the rows and each of the columns of the middle tray. What is a possible configuration of the nine bottles on the tray? Compare your solutions with each other.

![Figure 9](http://www.market4free.com/classifieds/ads/for-sale/antiques-vintage)

Figure 9: Light absorbed in a tray of 9 bottles by rows and columns.
For example, the amount of light absorbed in the first row is 5 units, so there could be two juice bottles and one empty bottle or one milk bottle and two empty bottles in the first row.

1. How many solutions are possible?
   There are four possible solutions. Notice that the two solutions in the second row are left-right reflections of the two solutions in the first row.

   ![Solution Grids](image)

2. What do we need in order to solve this problem (with just one solution)?
   We need more information. For example, if we know the sum of one (or more) of the diagonals, that might help. In this example, knowing the sum of the main diagonal will narrow the number of solutions to 1.

3. How many pieces of data are we trying to solve for?
   There are nine bottles, so there are 9 pieces of data we are solving for. So at most we would need 9 pieces of information.

4. How many pieces of information are we given?
   We were given 6 pieces of information – the sum of the 3 horizontal and 3 vertical lines. Again, note that one more piece of information (the sum of the diagonal) is enough to solve the problem.

5. Suppose you are told the amount of light absorbed through the diagonal going down from left to right is 6. Can you narrow down the possible solutions to the problem?
   Yes, the solution is

   ![Solution Grids](image)
Now, suppose that we are given different 2-dimensional views of a three dimensional object made of blocks. Can we reconstruct the object?

Activity 4: Reconstructing 3D objects with blocks.
For this activity, each group will need 7-8 small blocks. Given the top, front and right views of an object as shown in Figure 10, reconstruct with blocks what the 3D object looks like using 7 blocks. Is there a unique answer?

![Images of top, front, and right views of a 3D object with 7 blocks.](image)

Solution:

Here’s another one to try: Given the top, front and right views of an object in Figure 11, reconstruct what the 3D object looks like using 8 blocks.

![Images of top, front, and right views of a 3D object with 8 blocks.](image)
Solution:

Activity 5: Reconstructing 3D object with “floating” balls
You will need small styrofoam balls, long toothpicks, and a block of Styrofoam (or cardboard, toothpicks and gumdrops). We use these to represent a 3 by 3 by 3 cube (see the solution figures), so there are 27 possible positions a styrofoam ball could be in. Compared to the previous activity, there is an added complexity that the balls can “hang in mid-air.” With the block activity, for a block to be up, it had to be on another block. That is not the case here.

Consider the example in Figure 12 that shows the top, front and right views. Each shaded square indicates a styrofoam ball.

Figure 12: Different views of an object with 6 balls.

Solution: Here are pictures of the top, front and right solution.
Section 2 Homework:

1. Given the numbered grid and that there are 4 pennies, find the location of the pennies.

2. Given the numbered grid and that there are 5 pennies, find the location of the pennies. Is there a unique answer?
3. Milk/Juice Bottles:
   a) Determine 4 possible solutions to the milk/juice problem below.
   b) Find solution(s) if the main diagonal (the diagonal from top left to bottom right) absorbs 6 units.
   c) Find solution(s) if the off diagonal (the diagonal from top right to bottom left) absorbs 5 units.

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4. Given the top, front and right views of an object, reconstruct what the 7-block 3D object looks like.

5. Given the top, front and right views of an object, reconstruct what the 5-block 3D object looks like.
6. Use the styrofoam balls to reconstruct the 3D object that has the following top, front and right views.

![Top view](image1)

![Front view](image2)

![Right view](image3)

**Solutions to Homework for Section 2**

1. 

```
X
X
X
X
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2. It is unique

```
X
X
X
X
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3. a. The four possible solutions are

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1 1 2 4
2 3 3 8
3 1 2 6
6 5 7
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1 2 1 4
3 2 3 8
2 1 3 6
6 5 7
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4.

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6. Picture of solution from front, right and top
Section 3: Slices building to a 3-D Image – The Geometry of Reconstruction

Driving Question: How can 3-D images of an object be created given 2-dimensional shadows of the object?

This section begins with a quote and image from Plato’s Republic that emphasizes creating three-dimensional images from shadows. It continues with a closer look at the geometry of tomography, especially as applied to medical imaging. The geometry of tomography involves reconstructing 3D objects from 2-dimensional slices called profiles (or shadows).

The following quote is from Book VII of The Republic by Plato “The Allegory of the cave.”

"Behold! Human beings living in an underground den; here they have been from their childhood, and have their legs and necks chained so that they cannot move, and can see only before them, being prevented by the chains from turning round their heads. Above and behind them a fire is blazing at a distance, and between the fire and the prisoners there is a raised way; and you will see, if you look, a low wall built along the way, like the screen which
marionette players have in front of them, over which they show their puppets.”

Figure 13: Drawing corresponding to “Allegory of the cave” in Plato’s Republic

Can the prisoners always tell what the marionettes are holding? Does the angle that the object is held make a difference? What about the angle of the light? If you saw multiple shadows from different angles of the same object, would that help?

Computational Thinking Alert: What do you need to know – what makes a difference? What information is necessary to solve the problem?

In Figure 13, one of the shadows looks like a cat, but in reality the marionette is holding a cutout of a cat. If the cat cutout is held at a different angle, it would become clear that it was not a real cat. So, the angle of the object with the light does make a difference. And seeing multiple angles helps clarify what the object is.

When we take a classical x-ray, the object is positioned between a source of x-rays and a detector and a beam of x-rays penetrates the part of the body we are interested in, for example the brain or the shoulder bone. CT scans, however, reconstruct a 3-dimensional image by combining individual slices or profiles as shown in Figure 14. Each x-ray can be thought of as a profile, and the main idea for reconstruction is to use multiple profiles or measurements of the
amount of x-ray energy absorbed as reflected in the density of the image since
the higher the density the less energy that will be absorbed. We take these
measurements from various angles to reconstruct the image.

Figure 14: CT scan of a brain skull colored for clarity. (Wikipedia Commons)

Here are some examples of profile representations. In Figure 15, x-rays are
going through a square (top to bottom). The first profile representation is a plot
of the density. The density is 0 on the left and right and nonzero in the middle.
The second profile representation is like the x-ray we think of. The middle
section is white, corresponding to the higher density – like in an x-ray the bones
are white since they have the highest density.

Figure 15: Profile representations for a solid square
Try to sketch two profile representations for the square rotated 45° shown in Figure 16.

Solution:

Figure 16: Solid square rotated 45°
The density plot grows until the middle point because the distance the x-ray travels through the object grows. The second representation starts black, then fades to white, then fades back to black.

Try to sketch the two profile representations for the new shape shown in Figure 17.

Figure 17: New shape

Solution:
Notice that the density representation is the same height because the distance vertically to go through the object remains constant even though the middle part is offset.

**Homework for Section 3:**
1. Sketch the two profile representations for the cross.

![Cross Profile](image)

2. Sketch the profile representations for the V.

![V Profile](image)
Solutions:

1. 
   ![Diagram](image1)

2. 
   ![Diagram](image2)

Note here and in the example above that the distance through a section is not constant.
Section 4: Computational thinking revisited as a whole and an application to food safety

Driving Questions: How many slices do you need – how many can you get with today’s technology? How do you work with so much data, even in one slice? Are there nonhuman applications of tomography?

4.1 Data
Now that we know how to get profile representations from individual x-rays or slices (consider constructions from 3-D objects from 2-D representations). What else do we need to know to use CT-scans for diagnostic purposes?

There is a difference between what data can be obtained from standard CT-scans available to hospitals and other medical imaging centers and what can be obtained for research purposes from cadavers. The standard CT-scanners available at hospitals and imaging centers are able to collect data for 64 slices on one full rotation around the body. That is, there are 64 different detectors, one to get each slice. This size is significantly larger than the 2-slice CT-scans in 1994 and the 1998 4-slice CT-scans. The more slices a CT-scan can take in one revolution the more quickly and accurately the diagnosis can be made. One complete rotation minimizes the radiation compared to using multiple revolutions to get more slices. However, 64 slices produces a lot of data as each image consists of minimally 7.5 megabytes of data.

The U.S. National Library of Medicine at the National Institute of Health in 1994 launched the Visible Human Project to create digital image datasets of complete human male and female cadavers in MRI, CT and anatomical modes and make them available around the world. Data can be accessed on their website at http://www.nlm.nih.gov/research/visible/getting_data.html. Data and the site are updated yearly as new technology becomes available.

There is a lot of data! The CT data for the male cadaver consists of CT scans of the entire body taken at 1 mm intervals (it rotates 1mm, scans, then rotates 1mm and scans, etc). Each scan is at a resolution of 512 pixels by 512 pixels, while each anatomical image (from PET scans) is 2048 pixels by 1216 pixels. The total storage needed for 1 image (containing both CT and PET data) is 7.5 megabytes of data (equivalent to about 2 MP3 files). The anatomical cross-sections are also at 1 mm intervals and coincide with the CT axial images. There are 1871 cross-sections for each mode (CT and anatomy) obtained from the male cadaver. Thus, the total storage for 1871 cross-sections is 7.5*1871=14032.5 megabytes (equivalent to about 2339 songs).
The dataset from the female cadaver has the same characteristics as the male cadaver with one exception. The anatomical images were obtained at 0.33 mm intervals instead of 1.0 mm intervals. This results in over 5,000 anatomical images: making the female dataset about 40 gigabytes (over 10,000 songs). Spacing in the "Z" direction was reduced to 0.33 mm to match the pixel spacing in the "XY" plane. This alignment enables developers who are interested in three-dimensional reconstructions to work with 3-dimensional versions of pixels called voxels.

A voxel or (Volumetric Picture Element) is a volume element, which represents a value on a grid in 3-dimensional space. For example, the floating balls in section 2, activity 5 each represent a voxel. This is analogous to a pixel, which represents 2-D image data in a bitmap (which is sometimes referred to as a pixmap). Instead, the position of a voxel is inferred based upon its position relative to other voxels. Some volumetric displays use voxels to describe their resolution. For example, a display might be able to show 512×512×512 voxels just like a jpeg might show 512x512 pixels.

A voxel is particularly useful in CT-scans since a voxel represents a single sample, or data point, on a regularly spaced, three-dimensional grid. This data point can consist of a single piece of data, such as density of the substance at that point, or multiple pieces of data, such as a color in addition to density. Depending on the type of data and the intended use for the dataset, this missing information may be reconstructed and/or approximated. In CT-scans, the values are units that give the opacity (indicate the amount of energy absorbed) of material to x-rays. The picture shown in Figure 18 could be viewed as a group of voxels:

![Figure 18: A group of seven voxels](image)

The picture shown in Figure 19 is 3 of the 64 images (slices) taken of a 93-year old woman with a CT-scanner in 2006. The bones are the brightest, most white, as they absorb virtually no energy, are least opaque. What can you see?
4.2. An application to Food Safety

This subsection considers how the imaging techniques work to determine what is in the interior of an object, and provides a recent application of tomography.

A recent experimental application of CT scans is to food safety. Despite rigorous national standards for food safety, there continues to be incidents of E-coli-infected spinach and salmonella-infected tomatoes reaching the tables of consumers around the country. One example (from August-September 2011) is of melons from Colorado that carried listeria - a potentially deadly bacteria - across the United States, killing many. To help prevent such a spread of disease, food-engineering experts from Texas A&M are using CT scans to build disease and treatment maps of produce, that is, to first locate the common places on a melon where listeria grow, then use the CT scan to kill harmful bacteria, insects and parasites (see Figure 20). Special care is taken to determine the exact dose of ionizing radiation (the radiation from an x-ray) to expose fruits and vegetable to. Using computer simulations, they can take the scanned images of the fruit to compute the smallest dose of radiation to reach each part. No fruit or vegetable is exactly alike, so mapping out dosages is both challenging and amenable to CT scan technology. Proper mapping allows for
safe irradiation (exposure to controlled amounts of ionizing radiation for a specific time period to inactivate pathogens) of the produce.

In addition, it is possible to scan products such as strawberries to detect the presence of bacteria without harming the strawberries themselves.

Figure 20. An illustration of the procedure used to calculate the dose distribution in a whole cantaloupe. (Castell-Perez and Moreira, *Food Safety Magazine*, February-March 2011.)

Section 5: Final Assessment

1. The student should research a topic and write a 3-4 page paper on it. The paper should include an introduction and a conclusion, and contain a mathematical and technical discussion of the tomography. It should also include the connection to computational thinking. They may use some of the work they did in the first homework assignment as background.

   Possible projects include:
   - Magnetic resonance imaging (MRI)
   - Functional MRI
   - Electrical impedance tomography
   - Atom probe tomography
   - Optical tomography
   - Ocean acoustic tomography
- Food safety tomography (provide more detail than is here.)
- Concrete solidity tomography
- Tomography and geology

II. Exam
1. Describe how the yarn activity relates to tomography.
2. Give an example of tomography (excluding medical imaging).

3. Given the numbered grid, find the 5 hidden pennies.

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4. Given the top, front and right views of an object, reconstruct what the 8-block 3D object looks like.

   (a) top view  
   (b) front view  
   (c) right view

5. Given the top, front and right view of an object, reconstruct what the 3D object looks like using 9 balls (or gumdrops).

   top  
   front  
   right
6. Sketch the profile representations figure.

Solutions to Exam:

1. Tomography is determining internal structure. For the yarn activity, students are using a knitting needle to determine what kind of object is wrapped up in the yarn.

2. Answers vary and can be pulled from the module (food safety) or from the list of paper topics.

3. 

---

X

X

X

X

X

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4.

5.

TOP
Glossary

**CT (Computed Tomography or CT-scan):** imaging technique that combines a series of X-ray views taken from many different angles to produce cross-sectional images.

**MRI (Magnetic Resonance Imaging):** medical imaging technique that uses a large magnet to visualize detailed internal structures.

**PET (Positron Emission Tomography or PET scan):** imaging technique that helps visualize how the organs and tissues inside your body are functioning.

**Pixel:** (PICture Element) single point in a graphic 2-D image.

**Tomography:** the science of examining an internal structure with external measurements.

**Voxel:** (Volumetric Picture Element) is a volume element, which represents a value of a point on a grid in a 3-dimensional space image.

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## Appendix

**Checkerboard**

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