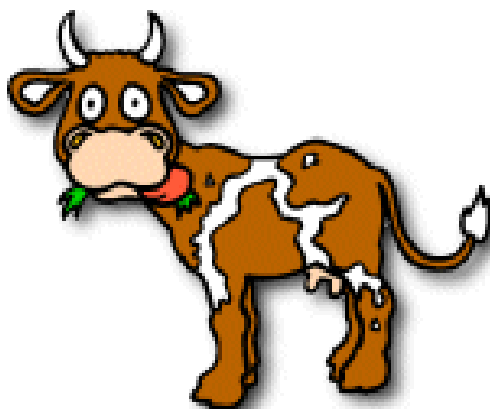




# **SELECTION OF EARLY WARNING TEST SITES FOR FOOT & MOUTH DISEASE IN DAIRY CATTLE**

## **A Bio-Surveillance Model**



**A CCICADA Module for Students in Classes 9-14**

**Teacher's Edition with embedded Student Edition**

**2011**

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## Module Overview

### *Synopsis of Module:*

The transmission of infectious diseases is a topic that is getting an immense amount of attention. One of the diseases on the list from the Center for Disease Control (CDC) is Foot and Mouth Disease (FMD), which can be contracted by cloven-footed animals and is highly contagious. If FMD is found in an area, it results in all herds within 10 km being slaughtered and financial devastation for the farmers involved. Although not currently in the United States, it is always just a plane ride away. Because of this (and because the author is from Wisconsin where dairy farms abound), this module was designed to educate students on the physical, economical, and moral issues surrounding FMD, as well as to use discrete mathematics to model farms in an area. The model is then used with a graph tool called “domination sets” to select a group of farms to serve as testing sites for early warning of an FMD infection.

Lectures, activities, and exercises are provided. However, the main focus of the module is a final group project that students work on outside of class. Part of it is done in homework assignment as the lesson progress. The students are required to do online research into FMD, and to apply lesson material to the completion of the final project. Both a paper and a presentation are highly recommended as a way to help students learn how to communicate their results and instill the lessons.

### *Topics:*

Foot and Mouth Disease in dairy cattle, FMD in other cloven-footed animals (if desired), bio-surveillance, discrete mathematics, graph theory, mathematical modeling

### *Target Audience:*

Grades 9 – 14

Students in classes involved with biology, agriculture/livestock, disease control, algebra I, algebra II, discrete mathematics, mathematics for liberal arts, and others

### *Length:*

The in-class material (including group work) will take about one week. This is two 75 minutes classes or three 50 minute classes. Students do some research outside of the classroom during the week and after the week. If the student groups are going to present, then an additional one or two class periods (depending on the number of groups) will be needed. It is suggested that each group receive 10 to 15 minutes to give their presentation.



# **NCTM *Principles and Standards for School Mathematics***

## **Geometry**

- Use visualization, spatial reasoning, and geometric modeling to solve problems

## **Problem Solving**

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems

## **Reasoning and Proof**

- Make and investigate mathematical conjectures
- Select and use various types of reasoning and methods of proof

## **Communication**

- Organize and consolidate mathematical thinking through communication
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others
- Use the language of mathematics to express mathematical ideas precisely

## **Connections**

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics to contexts outside of mathematics

## **Representation**

- Create and use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social, and mathematical phenomena

# National Science Education Standards

## Science as Inquiry

- Identify questions and concepts that guide scientific investigation
- Use technology and mathematics to improve investigations and communications
- Formulate and revise scientific explanations and models using logic and evidence
- Evaluate a solution and its consequences
- Communicate and defend a scientific argument

## Science in Personal and Social Perspectives

- Science and technology are essential social enterprises, but alone they can only indicate what can happen, not what should happen. The latter involves human decisions about the use of knowledge
- Understanding basic concepts and principles of science and technology should precede active debate about the economics, policies, politics and ethics of various science-and technology-related challenges. However, understanding science alone will not resolve local, national, or global challenges
- Individuals and society must decide on proposals involving new research and the introduction of new technologies into society. Decisions involve assessment of alternatives, risks, costs, and benefits, and consideration of who benefits and who suffers, who pays and who gains, and what the risks are and who bears them. Students should understand the appropriateness and value of basic questions – “What can happen?” – “What are the odds?” – and “How do scientists and engineers know what will happen?”

## *Goals and Objectives:*

### **A. MODULE Goals**

- Understand the need for early detection of infectious diseases in livestock to reduce loss of life and economic devastation
- Use introductory graph definitions to construct graph structures and answer questions related to a graph
- Model several applications, including a small number of dairy farms, using a graph
- Gather background information on FMD from online research
- Use minimum dominating sets as a tool to identify early detection test sites in the farming community they have been given to study

- Analyze the small model and consider what additional information is needed to a) make the model better, and b) bring results from other farming communities together to create testing sites over a larger area

## **B. LESSON Objectives**

### 1. **Lesson 1** (General Graph Concepts) Objectives:

The student will be able to

- find vertex and edge sets of simple graphs
- compute the degree of a vertex
- use the terms “vertex” and “vertices” correctly
- draw a complete graph
- find a uv-path in a graph

### 2. **Lesson 2** (Foot and Mouth Disease) Objectives:

The student will be able to

- understand the basic physical realities of FMD and what it does to livestock, its virulence, whether it is a fatal disease, and other aspects of the disease
- explain existing protocols for containing FMD in an area once it is identified
- determine the percentage of a herd that is infected once clinical signs of the disease are observed

### 3. **Lesson 3** (Mathematical Modeling) Objectives:

The student will be able to

- use a graph to model simple applications
- answer questions regarding an application using its representative graph

### 4. **Lesson 4** (Domination Sets) Objectives:

The student will be able to

- explain the difference between “minimal” and “minimum”
- when possible, find minimal dominating sets that are not minimum
- find all minimum dominating sets of a simple graph
- compute the minimum domination number of a graph  $G$

### 5. **The Project** Objectives:

The student will be able to

- model a community of dairy farms using a graph where edges are created using a given definition of “close”
- find minimum dominating sets of vertices of their graph, and identify the minimum domination number,  $\gamma(G)$
- relate information researched online regarding FMD to the graph representation of the dairy farms
- use dominating sets to select early detection testing centers for the community and explain/justify the selection
- list additional information that would be useful in providing a better selection of testing sites
- conjecture what would need to be done to incorporate all of the information from the different groups into one, cohesive testing site selection

## Notes to the Teacher

Welcome to this module that uses graph theory to help identify possible early warning testing sites for Foot & Mouth Disease. I hope you learn as much about the potential harm (both real and manufactured) that an outbreak of FMD would cause as I did. This is a great opportunity to learn how to apply a mathematical construct to a present concern. Hopefully, it will broaden the students' horizons regarding the disease aspects as well as introducing them to a "hands-on" type of mathematics.

This module is divided into four lessons. Each lesson should take about one 45-minute class period, and comes complete with activities and exercises. These lessons support the main focus of the module, which is building a model that represents a farming community and using it to select farms as early warning test sites for FMD. Thus, this module has a nice component of work outside of the classroom. **Notes to the teacher are given in red. They are not included in the student edition.**

- **Lesson 1** covers the basic definitions of graphs that the students will be using in the module. "Alerts" and "Questions" are highlighted to bring out some of the areas in which there can be confusion. This lesson will not be directly used in the project, but provides necessary background material on the mathematics that will be used later.
- **Lesson 2** brings in the topic for the project, which is Foot and Mouth Disease. There is plenty of information in the notes, but the main activity in this lesson will have students analyzing a chart on the spread of FMD in a 1,000-head dairy herd. After the in-class portion of this lesson, students are asked to do group research to find out more about FMD that will be used in their final project.
- **Lesson 3** will tie together Lesson 1 and Lesson 2. It is a section on mathematical modeling. Two simple, yet different, applications encompass the lesson. It is written to be interactive, so spaces are left for students to take notes, answer the questions, and build the models while you do the same on a text projector or overhead. It is at the end of this lesson that student groups will need to receive their collection of farms and use a graph to model them.
- **Lesson 4** contains the more specific graph theory the student will need to complete the project. Specifically, domination sets. Here the definitions are at a higher level than in Lesson 1. There are ample exercises on which the students can practice prior to using domination sets on the project. At the end of this lesson, the students should have enough background to be able to complete the project.
- **The project** is the focus of this module. It is included in Parts A, B, and C in Lessons 2, 3, and 4 respectively. In addition, the project in its entirety is included as part of the appendices at the end of the module. Although both a paper and presentation are highly recommended as outcomes for the module, one or the other can be used separately. It is



very different to give a presentation rather than to write a paper, and I find that students benefit from the opportunity to do both.

## Teacher Notes and Tips

### *Suggestions for Teaching this Module*

This module is a mixture of lecture, student activities and exploration outside of the classroom. Although it may seem that this may tie your hands, it actually gives you a lot of options for how you disseminate the information of each lesson. It is recommended that you use this guide along with the lessons as you prepare for each lesson.

I should note that the order of the lessons has been designed so that students can get part of their final project done in the order of the lessons. Please note that Lessons 1 and 2 may be interchanged if you want to give students more time to research FMD.

#### **To prepare ahead of time:**

Students will be working outside of the classroom on a specific set of farms that you have given the group. Therefore, it will help if you have already decided what you will be doing in regards to the farming community situation.

I have provided a county map of Wisconsin along with the name/town/county of 115 dairy farms in Wisconsin. If you want to use the included material, you will need to pick out 10 – 12 farms for each of the groups. I would make sure that there are some farms within those you pick out that are more than 10 km apart so the group has some variety. You do NOT have to figure out the mileage between each farm, as that is for the students to do. However, you can look up the county each farm is in and make an educated guess. You will also want some farms close together. Create enough groups of farms so that your students can work in groups of 3. If necessary, groups of 2 or 4 can be created. I have found that groups of 2 are either really good or really in need of motivation. Groups of 4 are either really good and cooperate well, or there are one or two people who think they can get away with doing nothing. Since there is a student evaluation as part of the project, this does not pay off, but that isn't always a good enough deterrent.

**\*\*If you want to personalize this project, you can try to [obtain the names of farms in your state](#).** I have found that this is not always information that is easy to retrieve. The Department of Agriculture can either be helpful or refuse to give what they consider private information. In that case, online you can look up farms that are for sale, farms that give tours, farming organizations that list some of their members, etc. in order to obtain enough farms so that each group has at least a few different ones.

#### **Lesson 1 (General Graph Concepts)**

**Notes:** This section gives definitions of graphs. Pay particular attention to whether students use the term “vertex” (one vertex) and the term “vertices” (more than one vertex)

correctly. They tend to say either “vertexes” or “verticie”. You will need to keep an open ear throughout the module! Also, note that when two edges cross, they do not form a vertex.

Another favorite mistake that students make is trying to draw an edge as: ●——

Every edge must have a vertex at each end, so this is not an edge.

**Format:** You can approach this lesson in one of two ways. One way is that you can lecture on the material prior to Activity 1 so the students are forced to take notes (this is my style), and then hand out the typed material when you hand out Activity 1. Or you can have the students read the material prior to Activity 1 on their own. In either instance, ask questions that emphasize the material in the “Alert” and “Question” boxes.

Once the questions and discussions have ended, pass out **Activity 1**. If students work together (which is a good idea), I suggest pairs for this initial assignment. If too many people get together, there is always someone who is left out. It shouldn’t take more than 5 minutes to complete Activity 1, and you can go over it in class.

Lesson 1 continues with a few more definitions. After “degree” is finished, try drawing another simple graph and see if the students can identify the degree of the vertices. Use that graph to also find uv-paths for different pairs of vertices. Notice that  $K_5$  has been left without edges for the students to fill in.  $K_6$  is an exercise.

**Homework: Exercise Set 1.** This set begins with a True/False concept check, and then goes on to other questions requiring both understanding of the definitions and construction.

If students are in the class and interested in programming, they can look up “adjacency matrices” online and then figure out a way to enter them into the computer (this is language dependent). After they have this ability, they can design a program to calculate the degree of each vertex.

**Alternative:** If you want to begin with FMD, Lesson 2 and Lesson 1 can be interchanged. That will give students more time with Part A of the project, which is doing some online research on FMD. Of course, that shouldn’t take too long, so does not need to be a consideration!

## Lesson 2 (Foot & Mouth Disease)

**Notes:** The information provided in the lesson is just to be used as motivation for the students beginning their online research. Their research will broaden their understanding (and hopefully, their interest) of FMD. Students are more comfortable researching material that is informational in nature than they are in researching mathematical topics. Therefore, they should be just fine given the parameters of the research.

I have not included photos of animals that have contracted this disease. It is ugly and I don’t like to see it, so I figure it is better that they find out this piece of information on their own! Also, many who have heard about FMD will automatically assume that all the animals die since it is so highly contagious. Not so. In their research, hopefully some students will find that only the very young, the infirm, and the very old will actually die from FMD. The rest may suffer from not

producing as much milk, etc. This presents a very real moral, economic and ethical situation upon which students can reflect. Different groups will come to different conclusions. Some will even find that in some countries, the animals are not slaughtered once they contract FMD. So, in addition to the information you give your students, enjoy what they find when they go out and learn more about this disease.

**Format:** This lesson lends itself well to handing out the entire thing to the students. Have them read up to Activity 2 and then form groups to work on it. **Activity 2** has been included in the middle of the lesson, but on separate pages in case you want to lecture on what is written and then have groups work on the activity. Go over Activity 2 in class after the groups are done (5 – 10 minutes depending on the depth in which the last questions are answered). Discussion on the last questions will aid the students when they begin writing their group papers.

After the discussion, wrap up with the remainder of the lesson. Either have the students read it on their own or lecture over it. Questions you can ask regarding this latter information are: “Why are vaccines not used even though they are available?” “What are the physical signs of the disease?” “In what ways is the disease transmitted?” “Why is it so hard to control the spread?”

There may be extra time after this lesson, so it may be a great time to have students pick their groups. If you have selected the groups, use this time to let them get together and plan. In any event, the groups will need to plan who will be doing what at the end of the class period.

**Homework:** Assign the **True/False Concept Check** and **Project Part A** that is at the end of the lesson.

### **Lesson 3 (Mathematical Modeling)**

**Notes:** This section contains two simple “real life” situations. Students can use some of what they learned in Lesson 1 to model these situations and then answer questions posed for them.

**Format:** THIS IS AN INTERACTIVE LESSON. It has been designed for use with either an overhead projector or a text projector where you and the students fill in the missing parts on the students’ handouts. You may want to let them work in groups on **Activity 3** and **Activity 4**, where they either design the graph or answer the questions or both. It is important that they understand the importance of how vertices and edges are assigned. It is often a good object lesson to have them try to assign things in opposite order and see if the question can be answered. For example, in the first activity, the vertices would have to be “get along” and “do not get along”. That means the edges would have to represent the cats. What a mess!!! The cats are not a relationship; getting along is the relationship, so they must comprise the edge set.

**Homework:** **Exercise Set 2** as well as **Project Part B** that is given at the end of the lesson.

## Lesson 4 (Domination Sets of Vertices)

**Notes:** Here is where the students learn what they need to use on their project. Domination set will be used on their graphs from Lesson 3 homework. The BIG lesson is the difference between “minimal” and “minimum”. It is not an easy concept for students to grasp. Hopefully, there are enough examples, activities and exercises so it is more understandable.

**Format:** There are important things that must not be misunderstood in this lesson, so I would suggest a lecture-style approach with questions and exercises. The important questions are actually in the lesson itself, and you need to be certain that they are noticed, answered, and understood. This will ensure a better level of student success as they finish their final project. Since 5 graphs are given in the exercises, you might want to use one or two of them in class for either additional lecture material or an activity (or one for both).

**Homework:** **Exercise Set 3** (those graphs not used in class), and **Project Part C**. This is a large portion of the project, and will require more time. It is getting the final paper and presentation ready, as well as applying the new concept of domination sets to finding potential test sites.

If there are students interested in programming who have done the activity listed earlier for Lesson 1, they might want to design a program to 1) find all the minimum domination sets of a graph  $G$ , and/or 2) find the domination number of  $G$ . Caution: This should only be done for very small graphs!

### The Project

- The project is designed to be done outside of the classroom, and is 75% based upon what the student learns in the module. The rest is to be done through online research, as well as analysis with group members where decisions must be made.
- Groups of 2 – 4 are advisable, with 3 being my favorite number. You may want to pick the groups, or allow self-selection.
- There is an evaluation component to the project where each student fills in an evaluation form (included in the appendices of this module) and turns it in. Since no one else besides the teacher sees it, I have found that students are much more honest than they would be otherwise. Give the project a grade, then distribute points as the evaluations (and your personal observations) warrant. For example, Susie, Tara, and Becky work on the project. It is quite apparent that Tara hasn't been doing much, and the evaluations bear that out—even though Tara thinks she has done 25% of the work. The project receives 47/50 points. Both Susie and Becky have come to see you and ask questions so you know they have done the vast majority of the work. They each get 47 points for the project. However, Tara gets only 37, a middle C, for her limited involvement.
- Give different groups different sets of farms. I find that 10 – 12 farms is a good number. It is too big to easily see an answer, but it isn't so big that it takes a super computer to find the dominating sets. Having different sets of farms makes things more interesting

when students present. Some will even have water between the farms, and students will hopefully read to see if FMD is transported over water.

- The project is built throughout the module, but additional time after the last lesson is completed will still be needed to do the last portion and to finalize the paper and/or presentation.
- Students should be responsible for gathering the background information. Do not feel that you need to have all the answers ahead of time. Actually, it is better if you do NOT have the answers so students have to depend on themselves and their groups to get this done.

### **Project Assessment**

- Assessment tools included with this module are: Guidelines for Groups, Rubric for Grading Presentations, and the Individual Assessment of Group.
- Technical paper
  - Grade structure, grammar, spelling, approach, as well as correctness of the mathematics (is the graph correct, are the dominating sets actually minimum, etc.). It would be a good idea to check on the graphs before Lesson 4, where they have to use them.
  - Should include figures, charts, and tables where appropriate, a reference page, and at least three outside references.
- Presentation
  - The rubric can be modified for what you look for, but gives an idea of what I look for in my class presentations.
  - Make sure the group's ability to disseminate information in the presentation is good—it should be different than the paper.
  - Encourage students to do something other than read from their papers! It is really boring!
  - Allow about 10 or 12 minutes for each presentation.
  - Everyone in the group should have some part in the presentation.
- Individual Assessment
  - Have everyone fill out an assessment form.
  - Use it to help assign grades.

### **Additional Reading if you want to know more:**

Graph Theory:

- Goodaire and Parmenter, *Discrete Mathematics with Graph Theory*, 2<sup>nd</sup> ed., Prentice Hall (2005).
- R. Grimaldi, *Discrete Mathematics*, 7<sup>th</sup> ed., Addison Wesley (2007).

Disease Surveillance:

- M. Salman (Ed.), *Animal Disease Surveillance and Survey systems, Methods & Applications*, Iowa State Press (2003).

Foot & Mouth Disease:

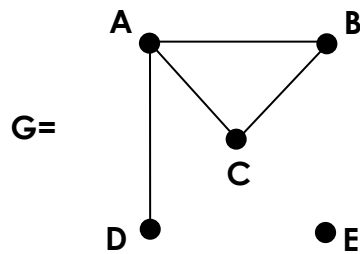
- M. Morpurgo, *Out of the Ashes*, Macmillan (2001).

- A. Woods, *A Manufactured Plague*, Earthscan Pub. (2004).

## LESSON 1: GRAPH THEORY CONCEPTS

Many things in our world and beyond can be described using mathematical objects. A graph is one of those objects, and is what we will be using throughout this module. So we begin our exploration by defining a graph and learning the language we need to use when talking about it.

A **graph**  $G$  is a nonempty set of **vertices** (objects that look like points) and a set of **edges** between the vertices. **Figure 1** gives an example of a graph  $G$ .



**Figure 1:** Graph  $G=(V,E)$  with  $V(G) = \{A,B,C,D,E\}$  and  $E(G) = \{AB,AC,AD,BC\}$

The set of vertices in  $G$  are labeled. We refer to this set as  **$V$**  or  **$V(G)$** . So for this graph,  $V(G) = \{A,B,C,D,E\}$  where  $A, B, C, D,$  and  $E$  are the **vertices of  $G$** .

**\*\*Alert:** Although the word “vertices” is used above for  $A, B, C, D,$  and  $E$ , it is not used if there is only one. If there is one, it is called a **vertex**. So,  $A$  and  $B$  are vertices of  $G$ , and  $A$  is a vertex of  $G$ .

Notice the lines in **Figure 1** between some pairs of vertices. These are the **edges of  $G$** . We refer to the set of edges as  **$E$**  or  **$E(G)$** . Now that we have the notation for edges, the formal definition of a graph  $G$  can be given.  **$G=(V,E)$**  is a graph  $G$  with a nonempty set of vertices  $V$ , and a set  $E$  of edges.

Every edge has a vertex at each end. Thus, when we write the name of an edge we use the labels of its vertices. There are several ways to write the name of an edge, and we will use the following convention in this module: *If there is an edge between vertices  $U$  and  $V$  in a graph, we write  $UV$  to represent that edge.* Therefore, for  $G$  in Figure 1,  $E(G) = \{AB, AC, AD, BC\}$ .

**\*\*Question:** Do you think we could write BA instead of AB? Would it make a difference? Is it a different edge?

Since there is no direction to the edges, edge AB can also be written as BA. It is the same edge.

For the edge AB, we say that:

- 1) A and B are **adjacent** vertices;
- 2) A is **incident** with edge AB, as is vertex B; and
- 3) AB is **incident** with vertices A and B.

Vertex E in **Figure 1** is not adjacent to any other vertex.

**\*\*Question:** What word or words can you think of that might describe the vertex E over by itself?

Students may say "alone", "lonely", etc. We are going for "isolated".

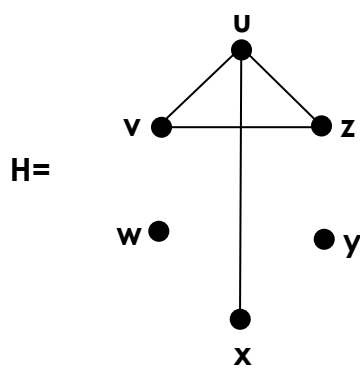
Any vertex that is not adjacent to another vertex is known as an isolated vertex.

**\*\*Alert:** When two edges cross in a graph, it does NOT create a new vertex. There is no vertex there unless one is drawn.

Now do **Activity 1** to check on your understanding of the concepts.

## Activity 1

Use graph H below in **Figure 2** to answer questions 1 – 6.



**Figure 2:** Graph H for use with **Activity 1**.

- Find  $V(H)$ . This must be a set using set notation.  $V(H)=\{u,v,w,x,y,z\}$
- Find  $E(H)$ . This must be a set using set notation.  $E(H)=\{uv,ux,uz,vz\}$   
Note that students may also use  $vu, xu, zu$  or  $zv$  for the edges.
- List the vertex or vertices adjacent to vertex  $z$ .  $u$  and  $v$
- Are there any isolated vertices? If yes, list them. If no, explain why.  
 $w$  and  $y$  are isolated vertices
- With which vertices is edge  $xu$  incident? Any edge is only incident with its end vertices. In this case,  $xu$  is incident with vertices  $x$  and  $u$ .
- Is vertex  $v$  incident with any edge? List any edge(s) with which  $v$  is incident. Vertex  $v$  is incident with edges  $vu$  and  $vz$ .



To continue with the definitions in Lesson 1, refer back to graph G in **Figure 1**. One of the aspects of a graph that is often important is the number of edges with which each vertex is incident. (Note that this definition is only good for simple graphs—those without multiple edges between vertices or a loop at a vertex.)

The **degree** of a vertex is the number of ends of edges with which it is incident. So the degree of A is 3, and we write **deg(A)=3**. Also,  $\text{deg}(B) = 2$ .

**\*\*Question:** What is  $\text{deg}(C)$ ,  $\text{deg}(D)$ , and  $\text{deg}(E)$ ?  $\text{deg}(C)=2$ ,  $\text{deg}(D)=1$ ,  $\text{deg}(E)=0$

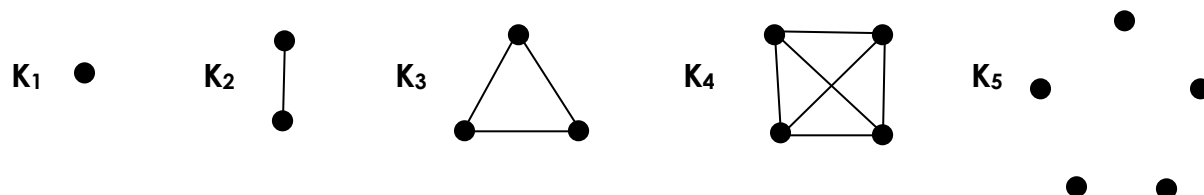
Note that an isolated vertex always has degree 0.

In a graph, it is often useful to be able to describe a way to get from one vertex  $u$  to another vertex  $v$ . Many times, we want the simplest way to get from  $u$  to  $v$ . Thus, we do not want to “visit” the same vertex more than once. This requires using what is known as a **path** from  $u$  to  $v$ , or a **uv-path**. In **Figure 1**, there are two paths from A to C. One is the edge AC, and we write the path using commas between vertices: A,C. The other AC-path uses edges AB and BC. This path is written as: A,B,C.

Notice that there is only one path from A to D: A,D.

**\*\*Question:** Why is there no other AD-path? A is repeated every time we try to go elsewhere. For example, path A,B,C has to come back through A to get to D.

For our last definition of this section, we have what is known as a **complete graph**. It is called “complete” because every two vertices have an edge between them—there is no other edge that can be added to the graph without repeating one. The notation for a complete graph on  $n$  vertices is  $K_n$ . Below are the first four complete graphs, and the fifth has been left for you to complete.



## Exercise Set 1

### 1. True/False concept check

- a. \_\_\_\_  $G=(V,E)$  must have at least one vertex. T
- b. \_\_\_\_  $G=(V,E)$  must have at least one edge. F
- c. \_\_\_\_ An edge can be drawn as  $\bullet$ — F
- d. \_\_\_\_ Edge CE can also be written as EC. T
- e. \_\_\_\_ When two edges cross, a vertex is formed. F

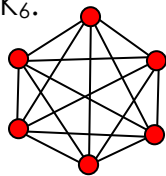
### 2. Fill in the blank.

- a. In **Figure 2**,  $w$  is an isolated \_\_\_\_\_. vertex
- b. In **Figure 2**,  $w$  and  $y$  are isolated \_\_\_\_\_. vertices
- c. The number of edges with which a vertex is incident is the \_\_\_\_\_ of the vertex. degree
- d.  $K_n$  is the \_\_\_\_\_ graph on  $n$  \_\_\_\_\_.  
complete, vertices

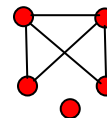
### 3. Using **Figure 2**:

- a. Find  $\deg(u)$ ,  $\deg(z)$ ,  $\deg(x)$ , and  $\deg(y)$ .  
 $\deg(u)=3$ ,  $\deg(z)=2$ ,  $\deg(x)=1$ ,  $\deg(y)=0$
- b. If possible, find two  $z$ - $x$ -paths. If impossible, explain why.  
 $z,u,x$  and  $z,v,u,x$

### 4. Draw $K_6$ .



5.a. possibility

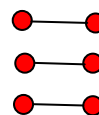


### 5. If possible, draw a graph:

- a. With 5 vertices where vertices have degrees 3, 3, 2, 2, 0.

5.b.

- b. With 6 vertices all of which have degree 1.

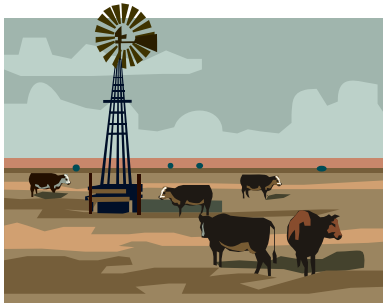


- c. With 6 vertices where 3 vertices have degree 2 and 3 vertices have degree 1. **Impossible.**

## LESSON 2: FOOT AND MOUTH DISEASE (FMD)

Many years ago, Foot & Mouth Disease was known as “Hoof & Mouth Disease”. You will still hear people call it that. This is because it is a disease contracted by cloven-footed animals such as cattle and sheep that have “cloven” hooves.

**\*\*Question:** What other cloven-footed animals can you think of? **Pig, goat, deer, etc.** Note that a horse does not. Although it has hooves, they are not cloven.



FMD is so contagious and is so difficult to detect early on, that people are very frightened of it. The Department of Homeland Security is very concerned about diseases like FMD being brought into the U.S. by terrorists. Although it has not been in the United States for decades, it is still found in the world. The Center for Disease Control (CDC) puts a high priority on keeping it out of the country. It is part of the biosurveillance that is done by the CDC . Any protocols for early detection are extremely useful, therefore, when they are developed.

So what is Foot and Mouth Disease? According to A. Woods:

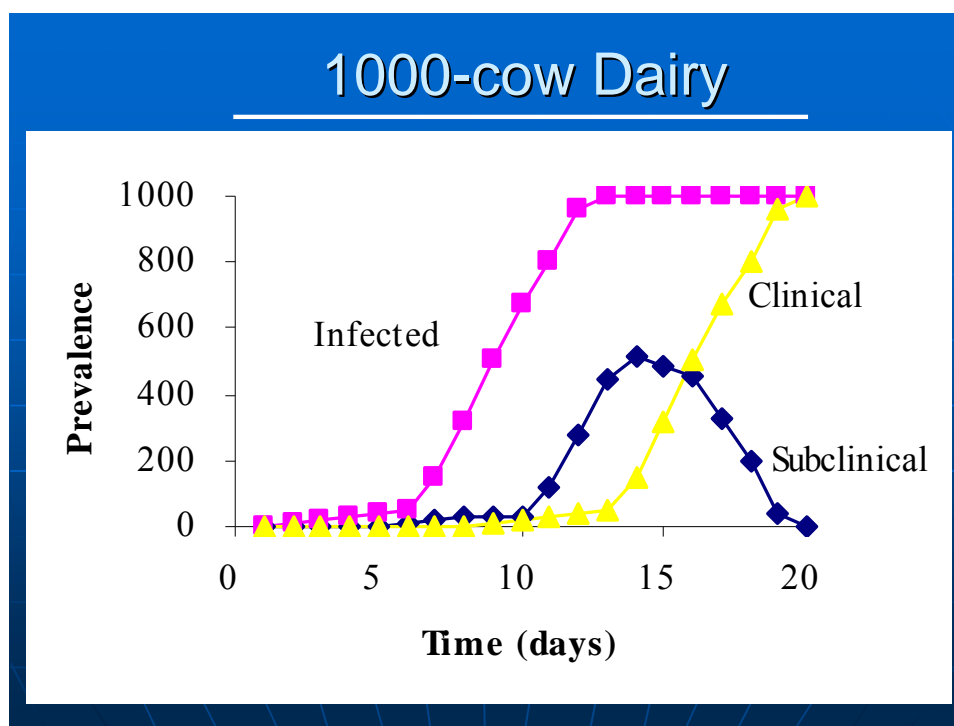
“Scientists say that it is a viral disease of cloven-footed mammals, which causes painful blisters to appear in the mouths and on feet and udders, and leads to lameness, drooling at the mouth, a loss of appetite and reduced milk production. They explain that, in the short term, many diseased animals will suffer. Some, especially the weak and the young, will die, and even animals recovering from the disease will show costly long-term reductions in growth rates and milk production. They also point out that FMD is one of the most contagious diseases know to man and can spread in almost every way imaginable.”<sup>1</sup>

In the preceding definition, note that the spread of the disease is mentioned. Actually, it is spread very easily, and a herd can be infected quickly. The spread

<sup>1</sup> A. Woods, *A Manufactured Plague*, Earthscan Pub. (2004).

can happen by animals being in near proximity (through air or water), dirt from an infected herd being tracked on a shoe or tire to another herd, and in many other ways. Once it has entered an area, it is extremely difficult to eradicate.

**Figure 3** shows the infection rate of a 1000-cow dairy. The pink line shows the number of infected cattle. The yellow line shows how many of the cattle are showing signs of the disease.

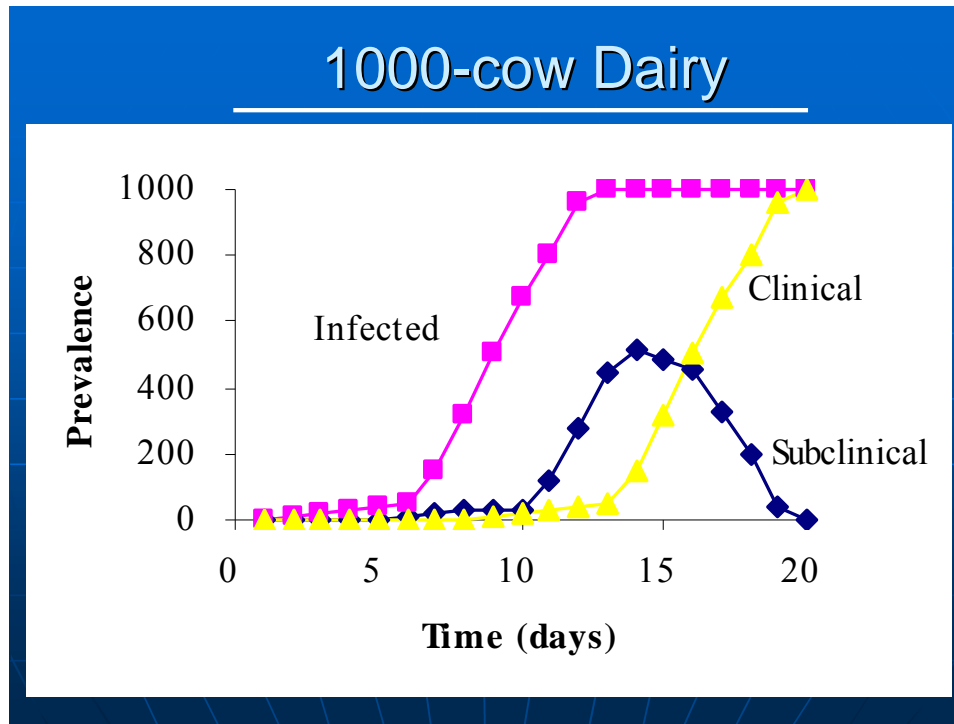


**Figure 3:** Graph showing the spread of FMD through a 1000-cow dairy herd.  
(Tim Carpenter, 2008)

Use **Figure 3** to answer the questions in **Activity 2**.

After you are done with **Activity 2**, finish the rest of the lesson.

## Activity 2



Work in groups of 2 or 3.

1. Using the yellow line, determine approximately how long it is before any cattle show symptoms.

It takes at least 10 days. The yellow line is just lifting off of the horizontal axis by day 10. That line is what we use to determine if there are any symptoms showing.

2. At the time the first signs of the disease show, approximately how many cattle have been infected (pink line)?

It is close to the entire herd. The pink line is almost at the top—it just depends on when students say the clinical signs are showing. In any event, there aren't many cows that have not been infected.

3. If the disease can be found using a blood test, would it be helpful in preventing the spread of the disease by checking the cattle's blood every so often?

Of course it would. Then it could be caught before it was clinically observed. The problems with being able to do that, however, are discussed in the following questions.

4. What might prevent a farmer, county agency, or U.S. agency from performing regular blood tests?

Students may be able to think of quite a few things. Some that I know of are a) cost, b) it takes too much time to draw blood from every animal, and c) the threat is deemed low since the disease hasn't been here for awhile.

5. What could be done to alleviate these problems?

Again, this will depend on what students have said above. Some solutions (or partial solutions) could be to test only a certain number of cattle per herd, test only certain herds that would represent an area (which is what students are setting up in their project), and other things that students will come up with.

Lesson 2 continued: Currently, blood tests are not reliable for detecting FMD. Portable sniffers can be used, but they are expensive and it takes a long time to test many cattle. Plus, the cattle have to already be infected for a couple of weeks for the sniffers to work. Thus, it is difficult to detect FMD before it infects an entire herd, but it is easy to bring the virus in.

How do the farmers feel about this?

“Farmers view FMD from a very different perspective (from the scientists). They say that it is one of the diseases they fear the most. It stops them from moving livestock around and sending them to market, restricts their social lives and leaves financial hardship in its wake. An unfortunate minority can recall the day when with a heavy heart, they rang the vet to report suspicious symptoms among their stock. They tell of their pain at seeing their animals slaughtered, a lifetime’s work destroyed; of the silence the next morning, and knowing that life would never be quite the same again.”<sup>2</sup>

So how is the disease controlled once it has been found? At the present (this is current as of 2001 when an epidemic in the United Kingdom broke out), every animal in an infected herd is slaughtered. In addition, the animals of all surrounding farms are slaughtered. Any movement of possibly infected animals to places such as slaughter houses, feed lots, 4-H competitions, county fairs, etc. is traced and the animals (those being traced and any herds with which they have had contact) are slaughtered.

The UK used a circle with a 10 km radius to contain an “infected” area. A circle with a 20 km radius was used to create the “surveillance zone” where animals were quarantined for three weeks to be certain they were not infected. Any cloven-footed animals within the 10 km circle were killed whether they were showing signs of the disease or not. You can see from **Figure 3** that if they had waited for the clinical signs of the disease to appear, it would be too late.

**\*\*Question:** Can the disease be cured? *Have students say what they think. Many will figure that since all the animals are being killed, it must kill everything it touches. It does not.*

<sup>2</sup> A. Woods, *A Manufactured Plague*, Earthscan Pub. (2004).

The sad truth is that yes, FMD can be cured. There is even a vaccine for it. The problem is that if you vaccinate a herd their blood tests will look like they have the disease. So farmers do not want to vaccinate because then they will not know if the disease is there. Plus, others will not want to buy the animals if the blood test shows positive for FMD, even though it is just the vaccine's presence.

A dairy cow that has had FMD may not produce as much milk as she did before. Beef cattle, pigs, and other meat animals still have perfectly good meat, but some countries (including the United States) will not buy meat or animal products from infected animals. You may think that it is because people can get the disease. However, no one has ever gotten FMD. Thus, entire herds and farmers' livelihoods are destroyed once the disease is found for reasons other than those we may normally associate with the need to stop a contagious disease.

Needless to say, being able to detect FMD as early as possible can lead to saving animal lives as well as having many monetary benefits for the farmers.

### **True/False Concept Check**

1. \_\_\_ FMD is a highly contagious disease. **T**
2. \_\_\_ If detected as soon as it clinically appears, part of the herd will probably not be infected if proper precautions are taken. **F**
3. \_\_\_ FMD directly kills most animals that contract it. **F**
4. \_\_\_ Cost effective ways exist to test cloven-footed animals for FMD. **F**



**Project—Part A** (To be given as homework after completing Lesson 2.)

Coordinate with others in your group to establish who is to do what. Be sure to do your part and get the information back to the entire group.

Use the Internet to research what you can find about Foot and Mouth Disease. Some information is more difficult to find than others. Include such things as:

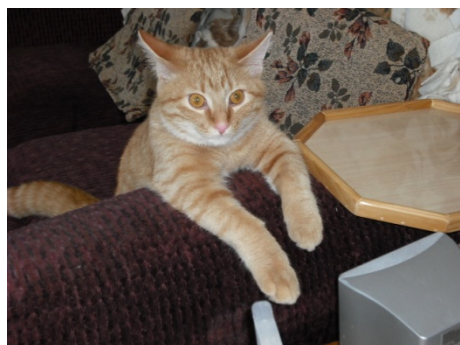
- 1) The protocols used by different countries in dealing with FMD once it is found. Are there any countries that do not automatically kill the animals?
- 2) Acquaint yourselves with the clinical signs of the disease. In other words, see if you can find photos of animals who have it, read about what it can do and what it does not do.
- 3) What you can find about vaccines and blood tests.
- 4) Other interesting facts that you find. Remember, these will go into a paper as background.

## LESSON 3: MATHEMATICAL MODELS USING GRAPHS

The first two lessons involved two completely different subjects. Yet in the final project, they will be used together. A graph will be employed to represent a collection of dairy farms.

In preparation for that, this section contains applications that show some ways graphs can model things in the real world. The presentation is a selection of examples, as is the exercise set.

### Example 1 (Activity 3)



Carol is a foster parent for several rescued cats who are waiting to find adoptive homes. Unfortunately, not all of the cats get along and Carol must find enough rooms in which to keep them so that the cats in each room get along. Having a small house, she would like to use as few rooms as possible.

If      Percy does not get along with Holly, Rommie or Ellie  
          Jackie does not get along with Holly or Ellie  
          Shady does not get along with Rommie  
          and Aggie gets along with everyone:



### How many rooms does Carol need?

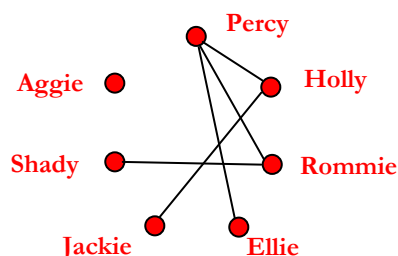
Sometimes the most difficult part of modeling is determining the most useful way to assign vertices and edges. You want to be able to make sense of the information you have and actually use it to get an answer to your question.

In this example, we let:

$$V(G) = \{ \text{cats} \}$$

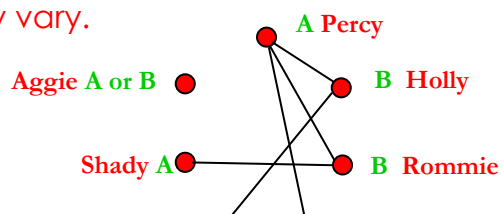
$E(G) = \{uv \mid \text{cat } u \text{ does not get along with cat } v\}$  (Note that this is read “the set of edges  $uv$  such that cat  $u$  does not get along with cat  $v$ ”.)

We will assume that if cat  $u$  does not get along with cat  $v$ , then cat  $v$  does not get along with cat  $u$ . Create graph  $G$ . Draw the vertices and label them with the cat names. Draw an edge between any two vertices that represent cats that do not get along. *Let students try their hand at this and then show them the finished version so they can check to make sure they are thinking correctly. It does not matter what order they have the vertices; only that the edges are between the correct labels.*



Let's ask some question about what we have here so we better understand the model.

- 1) If an edge is between two vertices, can the cats represented by those vertices be in the same room? **No—they do not get along**
- 2) If there is an isolated vertex, how does that fit in with what we are trying to do? **The cat represented by that vertex gets along with all other cats, so can be placed in any room.**
- 3) If we have a triangle in the graph (i.e.,  $K_3$ ), what is the minimum number of rooms we would need? **We would need 3, as none of the cats would get along with each other and would need to be in separate rooms.**
- 4) Redraw the graph from above. Using A, B, C, etc. to represent different rooms, label each vertex with one of them. Thus, if Percy's vertex is also labeled with an “A”, it means that Percy is to be placed in room A. Use as few rooms as possible. **Adjacent vertices must have different labels. It can be done using two rooms. Answers may vary.**

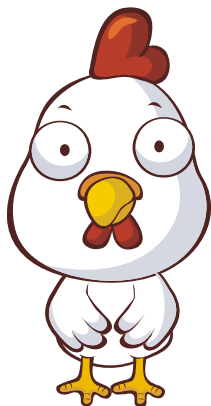


Jackie <sup>A</sup>      <sup>B</sup> Ellie

- 5) According to your answer to (4), what is the minimum number of rooms that Carol needs to house all of the foster cats so that they all get along in each room?      **2: rooms A and B**
- 6) How do you know that is the minimum number of rooms?

**It is impossible to do it with just 1 room since they do not all get along.**

### Example 2 (Activity 4)



Jarod had been out of work for quite awhile, and was thrilled when the new Chicken Chuckles restaurant that was coming to town hired him. Unfortunately, one of the first things he had to do was help with advertising the new restaurant. To do this, he had to put on a chicken costume and walk down all the streets of the surrounding neighborhood. Needless to say, Jarod only wanted to walk down each street once (if possible) and to get back to the restaurant as soon as he was able.

Perhaps we can help Jarod by figuring out a route for him to take where he walks down each street as few times as possible.

#### Street map of the neighborhood

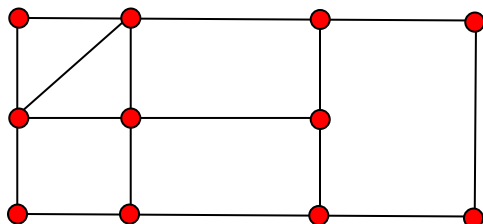


The first thing we need to do is use a graph to model the neighborhood. Usually, when modeling a street map, the following is used:

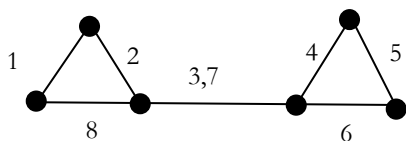
**$V(G) = \{\text{intersections and/or corners}\}$**

$E(G) = \{uv \mid \text{there is a street segment directly connecting intersection } u \text{ and intersection } v\}$

- 1) Draw a graph that models the street map. There is no need at this time to label the vertices.

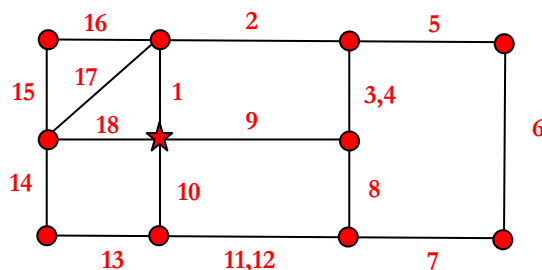


- 2) What we will do now is begin at the vertex where Chicken Chuckles is located (since that is where Jarod is) and create a route for him to take. Label each edge of the graph with the number of edge it is in the route. Some may need to be labeled more than once. For example:



Follow the edges in order from 1 to 2 to 3, etc., then the middle edge must be traveled again (it is the 3<sup>rd</sup> and 7<sup>th</sup> edge used) to get back to the beginning vertex.

Use this same idea to label the edges of the graph of the neighborhood that Jarod must walk. **Have a contest to see who can use every edge at least once and have the shortest route.** Remember to begin and end at Chicken Chuckles. *Answers will vary – possibly by quite a bit. This can be done using a route that is 18 edges long. Only two edges need to be repeated. There is no way to do it using 17 edges.*



This route can be given to Jarod for him to follow. He will only have to walk up two streets two times, and all the rest just once. He ought to be relieved!!

## Exercise Set 2

- 1) Finals are being scheduled. As always, two classes cannot have a final at the same time if there is at least one student they have in common. Sometimes, this is a real pain to figure out because there are so many classes. However, you are lucky because there are only a few that you need to deal with because all of the others have projects that are due instead of a test. Thus, these classes do not need to schedule a time for the final exam.

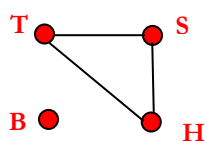
The table below shows some of the students and the classes for which you must schedule finals.

	Val	Juan	Omar	Sasha	Becky
Trigonometry		X		X	X
Spanish II	X			X	X
World History	X	X		X	
Cell Biology			X		

- a) Create a graph that models this situation.

Let  $V(G) = \{\text{classes}\}$

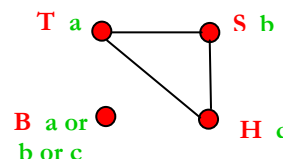
$E(G) = \{uv \mid \text{class } u \text{ and class } v \text{ share a student}\}$



Every column of the table that has more than 1 X indicates classes that share a student. Val's column gives us edge SH, while Juan's gives us TH. Omar's column does not create a conflict, so no edge is formed.

- b) If we are looking at times for finals, can two adjacent classes have the same time? Why? **No. Two classes with an edge between them have a common student. That student cannot take two finals at the same time.**
- c) Using a, b, c, etc. to represent different times for the finals, label each of the vertices with a time label. Use as few as possible.

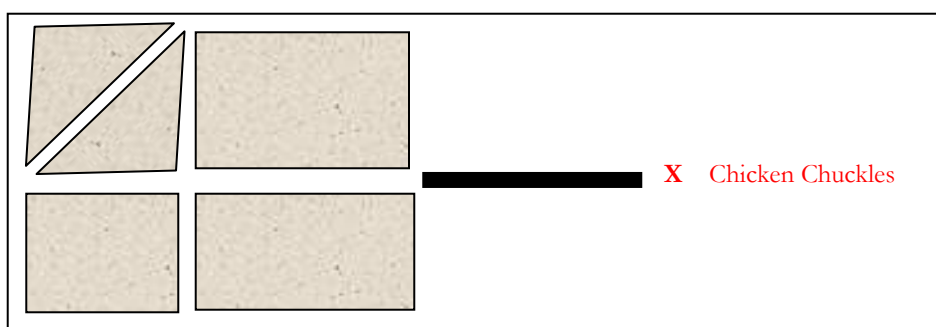
The fact that there is a triangle in this graph indicates there will need to be at least three times for the finals. As it turns out, it is possible to schedule the exams using three times (a, b and c) since isolated vertex B can go at any of the times.



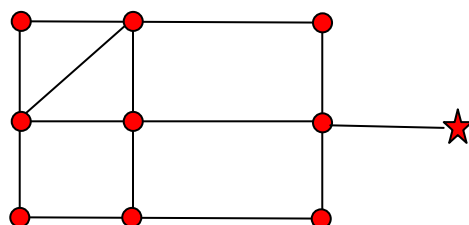
- d) What is the minimum number of time slots needed to schedule these four classes for finals? How do you know it is the minimum number?  
 Three time slots are needed to schedule the exams: a, b and c (whenever those are). It would be impossible to schedule the exams using two time slots because Trigonometry, Spanish II and World History each have common students. Those students cannot take two or three finals at the same time.

- 2) Jarod did such an excellent job, the owners of Chicken Chuckles want him to advertise in the area adjoining the one he did earlier. Use the map below to create the model for this exercise. Note that the thick line represents the one road through the neighborhood that Jarod will need to use when both leaving and coming back to Chicken Chuckles.

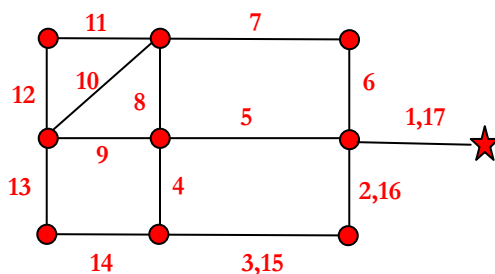
### Street map of the adjoining neighborhood



- a) Create a graph that models the street map.



- b) Beginning and ending at Chicken Chuckles, label the edges of the route. Use every edge at least once. Make the route as short as possible.



The shortest route possible uses 17 edges. In addition to the street from Chicken Chuckles being used twice, there will need to be two streets traveled twice in the new neighborhood. Routes will vary.



## **Project—Part B** (To be given as homework at the end of Lesson 3.)

- Groups will be given a list of dairy farms (or other farms) for this project.
  - If farms are in the same town, they should be counted as “close”.
  - If farms are at most 10 km apart, they should be counted as “close”.  
(Uncomfortable with kilometers? Look up online to see how many miles there are in 10 km and use miles!)
- 1) Go online and use a mapping tool to help you find the approximate distances between each pair of farms.
  - 2) For the farms in your regions, create a mileage table similar to that shown below. The mileage can be approximate.

<b>HERD A</b>	<b>HERD B</b>	<b>HERD C</b>
<b>HERD A</b>	<b>0</b>	<b>15</b>
<b>HERD B</b>	<b>15</b>	<b>0</b>
<b>HERD C</b>	<b>8</b>	<b>0</b>

- 3) Make a graph model of your group of farms. Use the mileage table to help you determine which farms are “close”.

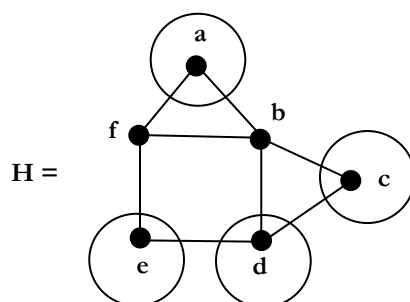
$$V(G) = \{\text{farms}\}$$

$$E(G) = \{uv \mid \text{farm } u \text{ and farm } v \text{ are “close”}\}$$

## LESSON 4: DOMINATING SETS OF VERTICES

In this final module, we learn about the tool that is going to be used on the graph model of your project. It requires some more graph theory definitions and for you to have some practice using it.

A **domination set** of a graph  $G$  is a set  $S$  of vertices. It is special because every vertex in  $G$  must either be in  $S$  or be adjacent to at least one vertex in  $S$ . Graph  $H$  in **Figure 4** below shows a graph with certain vertices circled. Those vertices have been chosen to be in set  $S_1$ . So,  $S_1 = \{a, c, d, e\}$ .



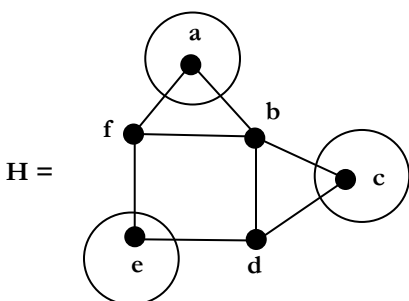
**Figure 4:** Graph  $H$  with the domination set  $S_1$  of vertices circled.

Notice that although vertices  $b$  and  $f$  are not in  $S_1$ , they are each adjacent to a vertex in  $S_1$ . (Make sure that student see that  $b$  is adjacent to  $a$ ,  $c$ , and  $d$  and  $f$  is adjacent to  $a$  and  $e$ , all in  $S_1$ .) Thus,  $S_1$  is a domination set of  $H$ .

**\*\*Question:** Can we remove any of the vertices of  $S_1$  making a smaller domination set of  $H$ ? The idea here is that we can remove  $c$  or  $d$  or  $e$  and still have every vertex in  $H$  adjacent to a vertex in  $S$ .

For this question, let's remove vertex  $d$  from  $S_1$  creating  $S_2 = \{a, c, e\}$ .  $S_2$  is a domination set because  $b$ ,  $d$ , and  $f$  are all adjacent to at least one of  $a$ ,  $c$ , or  $e$ .

**\*\*Question:** Given  $S_2 = \{a, c, e\}$ , can we remove any of the vertices from  $S_2$  and still have a domination set of  $H$ ?



**Figure 5:** Graph  $H$  with the domination set  $S_2$  of vertices circled.

Consider the vertices of  $S_2$  that are circled in **Figure 5**. If we remove vertex  $a$  from  $S_2$ , the vertex  $a$  will not be adjacent to either of the remaining circled vertices ( $c$  and  $e$ ). The same is true if we remove either  $c$  or  $e$  from  $S_2$ . So we cannot remove a vertex from  $S_2$  and still have a domination set of  $H$ . Because of this,  $S_2$  is called a **minimal domination set**.

**\*\*Question:** Is  $S_2$  the smallest domination set we can find for  $H$ ? Can we find a smaller one? *Students need to look at all the vertices, not just those in  $S_2$ .*

*Have students try to find a domination set using only two vertices. There are multiple sets that work:  $\{a, d\}$ ,  $\{d, f\}$ ,  $\{c, f\}$ , or  $\{b, e\}$ . They are smaller than  $S_2$  in that they contain fewer vertices.*

A domination set that uses the fewest number of vertices possible is called a **minimum domination set**. The number of elements in a minimum domination set for a graph  $G$  is called the **domination number of  $G$**  and is denoted  $\gamma(G)$  (that is, “gamma of  $G$ ”).

List a minimum domination set for graph  $H$  in **Figure 5** and calculate  $\gamma(H)$ .

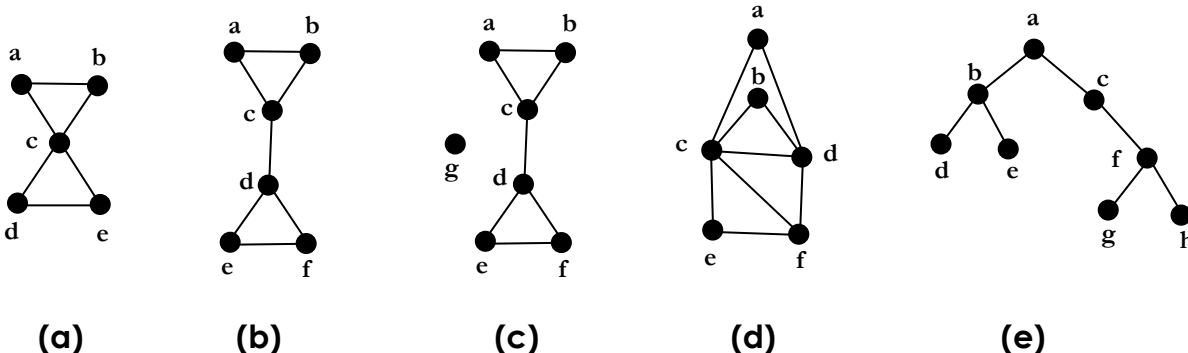
*Example:  $\{a, d\}$  and  $\gamma(H) = 2$*

**\*\*Question:** What is the difference between minimal and minimum? *Have students discuss this. Basically, minimal means we cannot make what we are given any smaller without losing the property we need (in this case, a domination set). Whereas, minimum means it has the fewest number of elements possible (in this case the number of vertices).*

## Exercise Set 3

Five graphs are given below. For each of them, answer the following three questions:

- 1) If possible, find a minimal domination set that is not a minimum domination set. If it is not possible, explain why.
- 2) Find a minimum domination set.
- 3) Determine  $\gamma(G)$ .



**(a)(1)**  $\{a,e\}$ ,  $\{a,d\}$ ,  $\{b,d\}$ , and  $\{b,e\}$  are all minimal, but not minimum domination sets because the removal of any vertex results in a set that does not dominate

**(a)(2)**  $\{c\}$  is the only minimum domination set

**(a)(3)**  $\gamma(G) = 1$  since there is only one element in the set in (a)(2)

**(b)(1)** There is no minimal domination set that is not also minimum. This is because any time we pick one of  $a$ ,  $b$ , or  $c$ , all of the three vertices are dominated and the same is true when picking  $d$ ,  $e$ , or  $f$ . In order to dominate all of the vertices, one of each of these must be chosen. If we add another vertex, it can be removed and the set of vertices will still dominate. For example, if I have  $S = \{a,b,d\}$ , all of the vertices are dominated. However, I can remove vertex  $b$  and still have a dominating set  $\{a,d\}$ . Thus,  $S$  is not minimal.

**(b)(2)** Any pair of vertices where one is from  $\{a,b,c\}$  and one is from  $\{d,e,f\}$  will work.

**(b)(3)**  $\gamma(G) = 2$

**(c)(1)** There is no minimal set that is not also minimum for the same reason as (b). The only difference is the isolated vertex  $g$ , which will need to be in any minimum set.

**(c)(2)** Any triple such as  $\{a,d,g\}$ , which has one vertex from  $\{a,b,c\}$ , one vertex from  $\{d,e,f\}$ , and  $g$ .

**(c)(3)**  $\gamma(G) = 3$

**(d)(1)** This is an interesting one, as there are minimal domination sets with different cardinality. For example, both  $\{a,b,f\}$  and  $\{d,f\}$  are minimal because we cannot remove any vertex from either set and still have a domination set. There are other possibilities as well. Just be sure the set cannot be reduced.

**(d)(2)**  $\{c\}$  because  $c$  is adjacent to all other vertices. Thus, it creates a domination set.

**(d)(3)**  $\gamma(G) = 1$

**(e)(1)** Trees (connected graphs that have no cycles), such as this graph, provide another opportunity to find minimal domination sets of a variety of sizes. In this graph, dominating sets such as  $\{a,d,e,g,h\}$  and  $\{b,c,g,h\}$  are both minimal since the removal of any vertex from either creates a set of vertices that no longer dominates. Other minimal sets also exist.

**(e)(2)**  $\{b,f\}$  is the only minimum dominating set

**(e)(3)**  $\gamma(G) = 2$

## **Project—Part C** (To be given as homework at the end of Lesson 4.)

- 1) Use the graph created in Part B for the following:
  - a. Find  $\gamma(G)$ .
  - b. Find all minimum domination sets.
  - c. Choose one of the minimum domination sets to serve as your choice of early warning test sites for FMD. The farms represented by those vertices will be the ones used for the testing. Be sure to use the background information and your own analysis to explain why you are choosing that particular domination set.
  
- 2) Group paper
  - a. Turn in a typed paper—one per group. The paper will be graded on content, correctness, ingenuity, spelling and grammar.
  - b. Use the information on FMD you researched in Part A in an introduction and throughout the paper as motivation for what is done and why.
    - a. Explain why as few sites as possible need to be used.
    - b. Explain why using domination sets helps select as few as possible given what we are considering as “close”.
  - c. Describe in detail the method used. Include your mileage table, graph, information on domination sets, and any other information you feel is pertinent to your paper. Justify what you do and decisions you make.
  - d. Make a recommendation as to which farms should be used as early warning testing sites based upon one of the minimum domination sets. Support your decision.
  - e. Create a conclusion. In that conclusion, include things such as:
    - a. Other information that you would need to be able to make better choices as to where the testing sites should go.
    - b. How recommendations from other farming communities (i.e., other groups in your class) could be combined with yours into a larger plan for the area. What problems do you face when you try to combine the results?
  - f. Include a reference page which has references for any of the information you are using that is not strictly from your own mind.

That includes all online sources used. There should be at least 3 from outside the class.

### 3) Presentation

- a. When making a presentation of the work you have done in a paper, remember that material needs to be included in a different way.
  - a. Use a lot of visual support—people like to look at pictures, so you don't have to put all of your words on something for others to read.
  - b. Include tables, figures, and other things you want to talk about. Keep notes by you so you don't forget what to say. This will keep the audience entertained, and you won't be reading the screen (if you use a PowerPoint presentation, overhead slides, or a text viewer).
- b. Everyone in the group should participate in preparing and giving the presentation.
- c. Prepare for a 10 minute presentation. Hit the highlights of the paper.
- d. Be prepared to answer questions from the class. Treat the presentation professionally, and treat each other with respect.

Remind students that they will be receiving an evaluation form to fill out for the project.

There were definitions given in Lesson 1, most notably “degree”, that have not been used directly in subsequent lessons. These were defined so the students would have some language to use in the project. I have found that students will pick domination sets where the vertices have the greatest degree. They are also able to talk about complete graphs when selecting one vertex from a group of vertices that are all adjacent. It is the ability to do this that makes a paper higher quality and shows the student is able to apply what they have learned.

## Appendix A

### Wisconsin County Map

Shown with the permission of Thompson Commmunications.



Source: [http://www.wicounties.org/WS\\_County\\_Directory.asp](http://www.wicounties.org/WS_County_Directory.asp)



## Appendix B

### List of 115 Dairy Farms in Wisconsin

The following list of farms with their towns and counties was compiled using a combination of online resources. Many of the farms are quite far apart. For each group, try to pick some farms that are close together and others that are farther apart. “Close” is being considered to be 10 km.

<b>FARM</b>	<b>Town/Village</b>	<b>County</b>
Jerian Holsteins	Barron	Barron
New Horizons B	Rice Lake	Barron
Morning Glory Dairy	DePere	Brown
Wayside Dairy	Greenleaf	Brown
Burnett Dairy	Alpha	Burnett
Faber	Chilton	Calumet
LaClare	Chilton	Calumet
Chilton	Chilton	Calumet
Hilbert	Hilbert	Calumet
E-8984	Boyd	Chippewa
Polzin	Cadott	Chippewa
Scientific Holsteins	Chippewa Falls	Chippewa
Chippewa Falls A	Chippewa Falls	Chippewa
Chippewa Falls B	Chippewa Falls	Chippewa
E-8979	Stanley	Chippewa
New Horizons H	Dorchester	Clark
F803	Hendren	Clark
Loyal-View & Royal-Mist	Loyal	Clark
New Horizons F	Loyal	Clark
512	Spencer	Clark
Holland Family	Thorp	Clark
507	Unity	Clark
Tetzner Dairy	Washburn	Clark
Wargo Acres	Lodi	Columbia
Bleu Mont	Blue Mounds	Dane
Hinchley's	Cambridge	Dane
UW-Madison Dairy Cattle	Madison	Dane
Femrite	Stoughton	Dane
Car-Bon	Beaver Dam	Dodge
Colfax	Colfax	Dunn
Bolen Vale	Downing	Dunn
Gingerbread Jersey	Augusta	Eau Claire
F801	Bridge Creek	Eau Claire

Homeland Dairy	Brandon	Fond du Lac
<b>FARM</b>	<b>Town/Village</b>	<b>County</b>
Rickert Brothers	Eldorado	Fond du Lac
Joas	Oakfield	Fond du Lac
Crystal Ball	Osceola	Fond du Lac
Pollack-Vu	Ripon	Fond du Lac
Vande Holsteins	Waupun	Fond du Lac
Alto	Waupun	Fond du Lac
New Horizons C	Fennimore	Grant
UW-Platteville Pioneer	Platteville	Grant
Sugar River	Albany	Green
Brodhead	Brodhead	Green
Decatur Dairy	Brodhead	Green
Blue Haven	Juda	Green
2002	Monroe	Green
Zimm-View	New Glarus	Green
Grand View	Berlin	Green Lake
Greenfield Hilltop	Markeson	Green Lake
Swanke's Dairy	Princeton	Green Lake
Blue Marble	Barneveld	Iowa
Grass	Dodgeville	Iowa
Gildale Holsteins	Hollandale	Iowa
Eachibon	Melrose	Jackson
Lundy	Jefferson	Jefferson
Dolph	Lake Mills	Jefferson
Crave Brothers Dairy	Waterloo	Jefferson
Rosy-Lane Holsteins	Watertown	Jefferson
Bumble B	Benton	Lafayette
2001	South Wayne	Lafayette
New Horizons D	Antigo	Langlade
5613	Merrill	Lincoln
Goehring's Clarks Mills	Cato	Manitowoc
Grotegut	Manitowoc	Manitowoc
Fitz-Pine	Newton	Manitowoc
Miltrim	Athens	Marathon
519	Dorchester	Marathon
New Horizons I	Marathon	Marathon
Marathon City	Marathon City	Marathon
New Horizons E	Marathon City	Marathon
Bernicks	Spencer	Marathon
Rosedale	Oxford	Marquette
Chapman Brothers	Tomah	Monroe
Dewgood Holsteins	Oconto	Oconto
Shady Lawn	Sobieski	Oconto
Lamers Dairy	Appleton	Outagamie
Caprine Supreme	Black Creek	Outagamie
Arla	Kaukauna	Outagamie
R-R Letters	Seymour	Outagamie

1878	Shiocton	Outagamie
<b>FARM</b>	<b>Town/Village</b>	<b>County</b>
Betzoldvale	Amery	Polk
1871	Amherst Junction	Portage
510	Kennan	Price
Davis	Kennan	Price
New Horizons G	Lone Rock	Richland
Larson Acres	Evansville	Rock
Janesville	Janesville	Rock
Honeycrest	Spring Valley	Rock
New Horizons A	Loganville	Sauk
Frozene	Westfield	Sauk
Drake Dairy	Elkhart Lake	Sheboygan
A-OK	Sheboygan Falls	Sheboygan
Elm Park	Sheboygan Falls	Sheboygan
Highland Crossing	Sheboygan Falls	Sheboygan
Ever-Green-View	Waldo	Sheboygan
492	Gilman	Taylor
513	Medford	Taylor
F206	Osseo	Trempealeau
Castle Rock Organic Dairy	Osseo	Trempealeau
Butler	Whitehall	Trempealeau
Wubbenhorst	Westby	Vernon
Krusen Grass	Elkhorn	Walworth
Van Dell	Sharon	Walworth
Sugar Creek	Elkhorn	Walworth
Springbrook Organic	Springbrook	Washburn
Cozy Nook	Waukesha	Waukesha
1858	Marion	Waupaca
Railane Holsteins	Scandinavia	Waupaca
1880	Plainfield	Waushara
Gateway-Acres	Poy Sippi	Waushara
White Clover	Menasha	Winnebago
Omro Dairy	Omro	Winnebago
Von Holzen	Winneconne	Winnebago
Majestic Oak	Marshfield	Wood

## Appendix C

### Group Work Guidelines

#### Team Work

- A. Make every effort to have team meetings when everyone can be there. If someone on the team habitually “cannot make it,” please see me.
- B. Everyone is expected to attend all team meetings.
- C. Participate in group discussions. When work is split up, be sure and have your portion done by the next meeting. If someone else needs to do it, you will not receive full credit for the project.
- D. Each member of each team is important. It is possible that someone on the team will see a solution before anyone else does. Let the other people also have input. There may be more than one way to view this.
- E. This is not a race.
- F. Group participation does not mean that everyone discovers every answer. But everyone should give a good amount of time to trying. There are only three weeks until papers are due and presentations will be given. In research, you must begin early. I will expect reports on your findings as the days go by.
- G. Each member of the group will have the opportunity (during the final exam) to evaluate the participation level of each member of their group (including themselves). It is important, therefore, that you do your best at all times, and that others know you are working on things.
- H. I am the complaint department. Bring personnel problems to me if they get too bad. Try to work them out as a group first.
- I. Treat each member of your group with respect. No one should be made to feel stupid or unwanted. Disrespect in any form is one thing that I absolutely will not tolerate. Act professionally.
- J. Above all, try to enjoy what you are doing. This is truly a unique experience. No one has done it before. Do your best!

## Appendix D

### Individual Evaluation of the Group

I make sure that students are able to fill these out and get them to me so that no one else sees them. Everyone has to fill one out if they want their project grade.

#### PROJECT EVALUATION SHEET

(consider all of the work done for the research, paper, and the presentation)

<b>NAMES OF GROUP MEMBERS (including yourself)</b>	<b>% of group work done (should add up to 100% for total in column)</b>	<b>Comments</b>
1. Self:		
2.		
3.		
4.		

## Appendix E

### Rubric for Group Presentations

When I have my groups do presentations, everyone has to be ready on the first day. We pick the group names out of a hat. The paper isn't due until the last day. Everyone is expected to be in class on every day there is a presentation. Missing one of the days lowers the presentation grade.

DESCRIPTION	POINTS POSSIBLE	POINTS EARNED	COMMENTS
<b>I. PRESENTATION</b>			
<b>A.</b> Audio: Can the class understand what you are trying to tell them? Do you say what you mean?	2		
<b>B.</b> Visual: Do you have poor, fair, good or excellent visual aids? Do they help the class understand what you are presenting?	2		
<b>C.</b> Interaction with the class and maturity of presentation.	1		
<b>II. PROBLEM</b>			
<b>A.</b> Understanding the problem: How well do you understand the problem you have been given? How well do you explain the problem to the class?	2		
<b>B.</b> Explanation of your approach to solving the problem, and how you got to your solution.	3		
<b>C.</b> Correctness of the mathematics and conclusion(s).	3		
<b>III. EVIDENCE OF TIME SPENT</b>			
<b>A.</b> On solving the problem (this is independent of whether you actually solved the problem or not).	1		
<b>B.</b> On preparing the presentation.	1		
<b>TOTAL</b>	<b>15</b>		

## Appendix F

### The Project

- Groups:** 2 – 4 students each, with 3 being a good number.
- Length:** Students should be able to complete the research within 3-5 days, do the modeling and analysis in 3-5 days, then write a paper and prepare a presentation. The total amount of time to allow is 2 – 3 weeks, depending upon whether presentations are given.
- Purpose:** Use graph theory to model dairy farm communities. Then use dominating sets of vertices to select a group of farms to serve as test sites for an early warning system to detect Foot & Mouth Disease.

#### Part A: What is Foot & Mouth Disease?

Use the Internet to research what you can find about Foot and Mouth Disease. Some information is more difficult to find than others. Include such things as:

- 1) The protocols used by different countries in dealing with FMD once it is found. Are there any countries that do not automatically kill the animals?
- 2) Acquaint yourselves with the clinical signs of the disease. In other words, see if you can find photos of animals who have it, read about what it can do and what it does not do.
- 3) What you can find about vaccines and blood tests.
- 4) Find other interesting facts, beliefs, etc. Remember, these will go into a paper as background.

#### Part B: The Graph Model

Student groups are to be given different sets of farms to use for their project. Be sure that there are 10 – 12 farms in each set.:

- If farms are in the same town, they should be counted as “close”.
- If farms are at most 10 km apart, they should be counted as “close”.  
(Uncomfortable with kilometers? Look up online to see how many miles there are in 10 km and use miles!)

- 1) Go online and use a mapping tool to help you find the approximate distances between each pair of farms.
- 2) For the farms in your regions, create a mileage table similar to that shown below. The mileage can be approximate.

	HERD A	HERD B	HERD C
HERD A	0	15	8
HERD B	15	0	0
HERD C	8	0	0

- 3) Make a graph model of your group of farms. Use the mileage table to help you determine which farms are “close”.

$$V(G) = \{\text{farms}\}$$

$$E(G) = \{uv \mid \text{farm } u \text{ and farm } v \text{ are “close”}\}$$

### Part C: Using Minimum Dominating Sets of Vertices

Use the graph created in Part B for the following:

- 1) Find  $\gamma(G)$ .
- 2) Find all minimum domination sets.
- 3) Choose one of the minimum domination sets to serve as your choice of early warning test sites for FMD. The farms represented by those vertices will be the ones used for the testing. Be sure to use the background information and your own analysis to explain why you are choosing that particular domination set.

### The Paper

- 1) Turn in a typed paper—one per group. The paper will be graded on content, correctness, ingenuity, spelling and grammar.
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  - a. Explain why as few sites as possible need to be used.
  - b. Explain why using domination sets helps select as few as possible given what we are considering as “close”.



- 3) Describe in detail the method used. Include your mileage table, graph, information on domination sets, and any other information you feel is pertinent to your paper. Justify what you do and decisions you make.
- 4) Make a recommendation as to which farms should be used as early warning testing sites based upon one of the minimum domination sets. Support your decision.
- 5) Create a conclusion. In that conclusion, include things such as:
  - a. Other information that you would need to be able to make better choices as to where the testing sites should go.
  - b. How recommendations from other farming communities (i.e., other groups in your class) could be combined with yours into a larger plan for the area. What problems do you face when you try to combine the results?
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## **Presentation**

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